

Entangling photons via the double quantum Zeno effect

Nicolai ten Brinke, Andreas Osterloh, and Ralf Schützhold*

Fakultät für Physik, Universität Duisburg-Essen, Lotharstrasse 1, 47057 Duisburg, Germany

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We propose a scheme for entangling two photons via the quantum Zeno effect, which describes the inhibition of quantum evolution by frequent measurements and is based on the difference between summing amplitudes and probabilities. For a given error probability P_{error} , our scheme requires that the one-photon loss rate $\xi_{1\gamma}$ and the two-photon absorption rate $\xi_{2\gamma}$ in some medium satisfy $\xi_{1\gamma}/\xi_{2\gamma} = 2P_{\text{error}}^2/\pi^2$, which is significantly improved in comparison to previous approaches. Again based on the quantum Zeno effect, as well as coherent excitations, we present a possibility to fulfill this requirement in an otherwise linear optics set-up.

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It is well known that photons possess many properties which make them very suitable candidates for quantum information processing. They can be well controlled and manipulated, as well as created and measured [1], even down to the single-photon level. Furthermore, photons can propagate over relatively long distances without significantly coupling to the environment. However, the latter advantage is also related to their main drawback – it is very hard to make two photons interact. Typically, before two photons interact by means of some non-linear medium, for example, at least one of them is absorbed. This motivates the idea to turn the problem around and to actually exploit the absorption in order to make photons interact. An indirect way of doing this is realized by the Knill-Laflamme-Milburn (KLM) gate [2] which induces interactions (probabilistically) via entangled ancilla photons and photo-detectors. This proposal has been studied in great detail theoretically (e.g., [3]) and experimentally (e.g., [4]).

A more direct way of exploiting the absorption of photons is to employ the quantum Zeno effect which describes the slow-down or even inhibition of quantum evolution by repeated measurements [5]. Imagine a quantum particle in a double-well potential initially confined to the right well, for example. After some time T , it would tunnel to the left well (and then back, etc). However, if we measure the position of the particle frequently, i.e., after very short time intervals $\Delta t \ll T$, it does not tunnel since each measurement projects the quantum state back to the right well. This is the basic picture of the quantum Zeno effect. Since the absorption of a photon in some medium is equivalent to measuring its position, a strong enough absorption probability can actually prevent the photon from tunneling/propagating into this medium. In contrast to earlier proposals for photon gates based on the quantum Zeno effect [6–8], our set-up is modified and so yields a significantly reduced error probability; see Eq. (2). Furthermore, we propose a realization based on free-space propagation, i.e., without waveguides, resonators, or optical fibers etc., which may induce losses and decoherence.

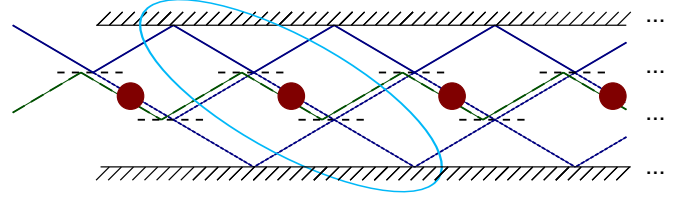


FIG. 1: Sketch of the macro-structure of the proposed set-up. Horizontal solid lines (top and bottom) denote perfect mirrors and horizontal dashed lines indicate beam splitters. The target photon (slanting dark blue line) enters the upper branch (initial state $|0\rangle$). The control photon (slanting dark green dashed line) enters the middle branch (if present, i.e., if in the state $|1\rangle$). Then both, the control photon and the part of the target photon which tunneled through the beam splitter into the middle branch pass the two-photon absorbing medium (brown circle). If the target photon is not absorbed, it may continue to tunnel to the lower branch through the second beam splitter. Each segment (tilted light blue oval) consists of an absorbing medium and two beam splitters.

The macro-structure of our set-up is sketched in Figure 1. It consists of perfect mirrors and beam splitters with reflectivity $\cos \epsilon$ and transmittivity $\sin \epsilon$ where $\epsilon \ll 1$. The target photon enters either the upper branch $|0\rangle$ or the lower branch $|1\rangle$. The control photon, on the other hand, enters the middle branch if it is in the state $|1\rangle$. The polarization and/or frequency of the control photon is chosen such that it is perfectly reflected by the beam splitters and thus stays in the middle branch. In the middle branch, there are absorbers with survival amplitude $e^{-\xi}$ where ξ assumes the value $\xi_{2\gamma}$ if the control photon is present and $\xi_{1\gamma}$ otherwise. Thus, after N segments, the initial $\vec{\psi}_{\text{in}}$ and final $\vec{\psi}_{\text{out}}$ amplitudes of the target photon are related via

$$\vec{\psi}_{\text{out}} = \begin{bmatrix} \cos \epsilon & -\sin \epsilon & 0 \\ e^{-\xi} \cos \epsilon \sin \epsilon & e^{-\xi} \cos^2 \epsilon & -\sin \epsilon \\ e^{-\xi} \sin^2 \epsilon & e^{-\xi} \cos \epsilon \sin \epsilon & \cos \epsilon \end{bmatrix}^N \cdot \vec{\psi}_{\text{in}}. \quad (1)$$

Now the main idea is the following: If the control photon is absent, absorption is very small $\xi_{1\gamma} \rightarrow 0$ and

the target photon which is initially in the upper branch $\vec{\psi}_{\text{in}} = (1, 0, 0)^T$, ends up in the lower branch with high probability $\vec{\psi}_{\text{out}} \approx (0, 0, 1)^T$, and vice versa. In the presence of the control photon, on the other hand, absorption is much stronger, $\xi_{2\gamma} \gg \xi_{1\gamma}$, and thus the target photon stays in the upper or lower branch, respectively, due to the quantum Zeno effect. The above scenario describes the ideal case – if the target photon is absorbed or does not end up in the designated branch, the set-up fails. The total probability for this failure is called error probability P_{error} and should be below a given threshold.

Since the number N of segments is related to the error probability via $P_{\text{error}} \geq \mathcal{O}(1/N)$, we consider the limit $N \gg 1$. In this limit, the desired behavior can be achieved [9] by setting $\epsilon = \pi/(\sqrt{2}N)$, $\xi_{1\gamma} = 2P_{\text{error}}/N$, and $\xi_{2\gamma} = \pi^2/(NP_{\text{error}})$. As a result, the two-photon absorption rate $\xi_{2\gamma}$ (with control photon) and the one-photon loss rate $\xi_{1\gamma}$ (without control photon) have to satisfy the constraint [9]

$$\kappa := \frac{\xi_{2\gamma}}{\xi_{1\gamma}} = \frac{\pi^2}{2P_{\text{error}}^2} \gg 1. \quad (2)$$

For the set-up discussed in [6], the corresponding error probability would be $P_{\text{error}} = 4N\xi_{1\gamma} + 2\pi^2/(N\xi_{2\gamma})$ and thus the required value for κ would be 64 times larger than in Eq. (2). This is due to the fact that the absorbing medium is only in the middle branch in our set-up. In Eq. (2) we assumed that the one-photon loss rate of the control photon is much smaller than that of the target photon. This could be achieved by choosing the detuning of the control photon (see below) to be much larger than that of the target photon. If the two loss rates were equal, for example, the necessary ratio κ increases [9] by a factor of 25 (which is still less than 64).

The above requirement (2) represents a major challenge, since two-photon processes are typically much weaker than one-photon effects. There have been proposals to induce strong two-photon absorption in resonators, cavities, or fibers [10], for example. Of course, these ideas could be applied to our set-up in Fig. 1 as well. However, in the following, we shall focus on free-space propagation which offers some advantages compared to wave-guides (but also has drawbacks, of course).

In order to calculate (2) for a concrete example, let us consider photons interacting with a single three-level system sketched in Figure 2(a). Starting from the usual non-relativistic Hamiltonian [11] ($\hbar = c = \epsilon_0 = \mu_0 = 1$),

$$\hat{H} = \frac{1}{2m} \left(\hat{\mathbf{p}} + q\hat{\mathbf{A}}(\hat{\mathbf{r}}) \right)^2 + V(\hat{\mathbf{r}}), \quad (3)$$

standard second-order perturbation theory [12] yields the two-photon absorption probability [9]

$$P_{2\gamma} = \frac{4\alpha_{\text{QED}}^2}{\pi^2\omega_1\omega_2} \frac{E_{12}^2 E_{23}^2}{\Delta^2} \frac{\ell_{\text{atom}}^4}{A^2}, \quad (4)$$

where α_{QED} is the fine-structure constant, and ω_1, ω_2 are the frequencies of target and control photon, respectively. Here $E_{12} = E_2 - E_1$ and $E_{23} = E_3 - E_2$ are the level spacings, while $\Delta = E_{12} - \omega_1$ is the detuning (of the target photon). Finally, A denotes the cross section area of the two photon wave-packets (we assume perfect spatial overlap), and ℓ_{atom} stands for the typical dipole (coupling) length of the atomic transitions. Note that the probability (4) does not depend on the length of the photon wave-packets (we assume perfect resonance $\omega_1 + \omega_2 = E_3 - E_1$); only the transversal dimensions (i.e., A) enter. In the optical regime, the contribution of the quadratic term $q^2 \mathbf{A}^2(\mathbf{r})$ to the probability (4) can be neglected.

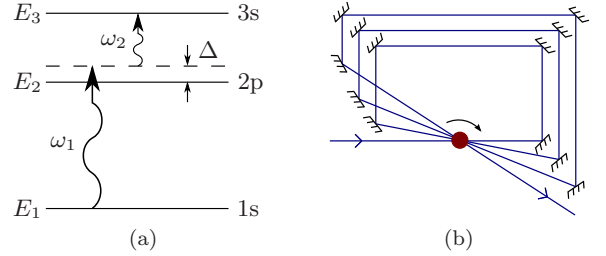


FIG. 2: Sketch (not to scale) of (a) the level scheme and (b) the internal structure of the absorbers where the two photons are sent through the same absorbing medium repeatedly. In order to ensure that the incident angle of the photons remains the same, the medium could be rotated.

Apart from the two-photon absorption probability (4), we obtain scattering of single photons by the three-level system at the same order $\mathcal{O}(\alpha_{\text{QED}}^2)$ of perturbation theory. In this process, the two contributions – second order in $q\mathbf{p} \cdot \mathbf{A}(\mathbf{r})$ and first order in $q^2 \mathbf{A}^2(\mathbf{r})$ – are of comparable strength in general. Under certain circumstances, such as special values of the frequencies $\omega_1 = \omega_2 = E_{12}\sqrt{1 - 2m\ell_{\text{atom}}^2 E_{12}}$, one could achieve destructive interference between the two contributions leading to $P_{1\gamma} \ll P_{2\gamma}$. However, in the following, we shall assume that the detuning Δ is very small (but still much larger than the natural line width of the middle 2p level). In this case, the $q^2 \mathbf{A}^2(\mathbf{r})$ can again be neglected and the scattering effect from $q\mathbf{p} \cdot \mathbf{A}(\mathbf{r})$ will be the dominant single-photon loss mechanism. As a result, the ratio between the two-photon absorption probability and the one-photon scattering probability scales as [9]

$$\frac{P_{2\gamma}}{P_{1\gamma}} = \frac{1}{\mathcal{O}(\pi\omega_{1,2}^2 A)}. \quad (5)$$

In view of the diffraction limit, this ratio is smaller than one and thus incompatible with the requirement (2).

In order to overcome this difficulty, we again exploit the quantum Zeno effect by sending the two photons n times through the *same* absorbing medium; see Fig-

ure 2(b). If the optical path length L in between is related to the sum of the two photon wave-numbers via $(k_1 + k_2)L \in 2\pi\mathbb{N}$, then the two-photon absorption amplitudes add up coherently. In contrast, assuming that this length L is larger than the size of the photon wave-packets, the scattering process behaves incoherently, i.e., like repeated measurements, and there only the probabilities add up. In this way, we can enhance the total two-photon absorption probability by a factor of n^2 compared to the one-photon scattering losses, which scale with n . Furthermore, possible local one-photon absorption effects do also violate the phase matching requirements because $k_{1,2}L \notin 2\pi\mathbb{N}$ and thus their probability does not scale with n at all. In this way, the requirement (2) could be achieved by sufficiently large n ,

$$\kappa = \frac{\xi_{2\gamma}}{\xi_{1\gamma}} = \frac{n}{\mathcal{O}(\pi\omega_{1,2}^2 A)}. \quad (6)$$

As an additional option, let us consider the coherent excitation (i.e., Dicke states [13], as often investigated regarding collective spontaneous emission, also known as Dicke super-radiance [14]) of a large number of atoms or molecules. After integrating out the middle (2p) level resulting in an effective coupling constant g , cf. Eq. (4), the effective Hamiltonian of S atoms or molecules at positions \mathbf{r}_ℓ reads

$$\hat{H} = g\hat{a}_1\hat{a}_2 \sum_{\ell=1}^S \sigma_\ell^+ \exp\{i\mathbf{r}_\ell \cdot (\mathbf{k}_1 + \mathbf{k}_2)\} + \text{H.c.}, \quad (7)$$

where $\sigma_\ell^+ = |3s\rangle\langle 1s|$ is the ladder operator for ℓ th atom and \hat{a}_1, \hat{a}_2 are the annihilation operators of the control and target photons with wave-numbers $\mathbf{k}_1, \mathbf{k}_2$, respectively. In terms of the effective spin- S operators $\Sigma^\pm = \Sigma^x \pm i\Sigma^y$ generating the $SU(2)$ algebra [15], the effective Hamiltonian can be written as $g\hat{a}_1\hat{a}_2\Sigma^+ + \text{H.c.}$ As a result, the transition matrix elements increase with the number s of excitations, $\Sigma^+|s\rangle = \sqrt{(S-s)(s+1)}|s+1\rangle$. Assuming $s \ll S$, the two-photon absorption probability scales with the number S of atoms or molecules times the number s of excitations – whereas other processes, which do not satisfy the phase matching conditions, only scale with S and thus are suppressed for $s \gg 1$. There is only one single-photon process [9] which does also scale with sS , namely, the absorption of, say, the target photon with \mathbf{k}_1 followed by the emission of a higher energetic photon with $2\mathbf{k}_1 + \mathbf{k}_2$. Fortunately, however, this process does not scale with $1/\Delta^2$ as in Eq. (4), since the “virtual” intermediate state is far from the middle (2p) level, and thus can be suppressed for small detuning.

The coherent state $|s\rangle$ could be sustained by two pump lasers with wave-numbers $\mathbf{k}'_1, \mathbf{k}'_2$, as long as they satisfy $\mathbf{k}'_1 + \mathbf{k}'_2 = \mathbf{k}_1 + \mathbf{k}_2$ and $\omega'_1 + \omega'_2 = \omega_1 + \omega_2$, in order to fulfill the same spatial and temporal phase matching conditions as the control and target photon. Their intensities I_1, I_2

are related to the ratio s/S via [9]

$$\frac{s}{S} = \left(\frac{4\pi\alpha_{\text{QED}}E_{12}E_{23}\ell_{\text{atom}}^2}{\omega'_1\omega'_2(\omega'_1 + \omega'_2)\Delta'} \right)^2 I_1 I_2. \quad (8)$$

However, the detuning Δ' cannot be chosen too small in order to avoid unwanted excitations of the middle (2p) level, $\Delta' \gg \sqrt{4\pi\alpha_{\text{QED}}I}\ell_{\text{atom}}$. With a typical dipole length of six Bohr radii, $\ell_{\text{atom}} = 6a_B$, and for a large but possibly realistic intensity of $I = 10^{10} \text{ W/cm}^2$, this translates into $\Delta' > 10^{14} \text{ Hz}$.

Let us insert some example parameters. We assume that target and control photon are in the optical regime (say around 500 nm) and adjust the detuning of the target photon to be $\Delta = 3 \cdot 10^{12} \text{ Hz}$, which is several orders of magnitude larger than the typical natural line width of the middle (2p) level (below GHz). Choosing the detuning of the control photon to be an order of magnitude larger, i.e., $3 \cdot 10^{13} \text{ Hz}$, the loss rate of the control photon can be neglected. For three different values of the error probability P_{error} , the Table I shows possible realizations of the set-up in Figure 1 with a number N of segments – where each segment has a two-photon absorption probability $P_{2\gamma}^{\text{segm}}$ and a one-photon loss probability $P_{1\gamma}^{\text{segm}}$, which correspond to the ratio κ . Assuming a focus at the diffraction limit $A = (\lambda/2)^2$, the last column gives the number n of repetitions necessary to reach this ratio κ via the mechanism sketched in Figure 2(b). Combining this enhancement mechanism with the coherent excitation of a large number s of atoms or molecules, the number in this column then corresponds to ns .

P_{error}	N	$P_{2\gamma}^{\text{segm}}$	$P_{1\gamma}^{\text{segm}}$	$\kappa = \xi_{2\gamma}/\xi_{1\gamma}$	n or ns
50%	10	95%	23%	12	635
25%	25	95%	4%	75	3 970
10%	60	98%	0.5%	760	40 231

TABLE I: Example values of the number N of segments, the two-photon absorption probability $P_{2\gamma}^{\text{segm}}$ and the one-photon loss probability $P_{1\gamma}^{\text{segm}}$ per segment, the corresponding ratio κ , as well as the enhancement factor n or ns necessary for reaching this ratio.

Note that these are example values with some margin for trade-off. For instance, $P_{\text{error}} = 50\%$ can also be reached by a ratio $\kappa = 9$ if we increase the number of segments to $N = 22$. This would lower the required number for n or ns down to 475. To see which values of s might be realistic, let us insert a detuning of the pump beam of $\Delta' = 3 \cdot 10^{14} \text{ Hz}$ into expression (8), where we obtain an excitation ratio of $s/S = 2 \cdot 10^{-5}$. If we imagine a glass plate of 10 μm thickness, for example, a volume with a cross section area of $A = (\lambda/2)^2$ would contain about $2 \cdot 10^{10}$ molecules. If one percent of these molecules is optically active, we get $S = 2 \cdot 10^8$, and thus $s = 4\,000$. By inserting the aforementioned values for the detuning of the target photon ($\Delta = 3 \cdot 10^{12} \text{ Hz}$) and

the control photon ($3 \cdot 10^{13}$ Hz), the two-photon absorption probability in (4) would be around $P_{2\gamma} = \mathcal{O}(10^{-11})$, which roughly fits to the value of $S = 2 \cdot 10^8$ discussed above.

Going to the limit, we may imagine increasing these two detunings to $\Delta = 3 \cdot 10^{13}$ Hz for the target photon and $3 \cdot 10^{14}$ Hz for the control photon (this value is in the infra-red region). In this case, the two-photon absorption probability in (4) is two orders of magnitude lower and we could use a glass plate of 1 mm thickness, which increases the maximum number s of excited atoms or molecules by two orders of magnitude. The values in Table I remain the same with the exception of the last column, where the required numbers for n or ns increase by 24%.

For the amplification mechanism sketched in Figure 2(b), a very tight focus is desirable. For the other enhancement mechanism based on Eq. (7), however, the spatial phase matching conditions become problematic if we focus down to the diffraction limit $A = (\lambda/2)^2$ due to the uncertainty in \mathbf{k} . Therefore, let us consider increasing the cross section area A . On the one hand, this would decrease the ratio (5) even further – but, on the other hand, the number S of atoms or molecules within this area A grows by the same factor. If we keep the pump laser intensity constant, the enhancement factor s compensates the shrinking ratio (5), i.e., a weaker focus with smaller uncertainty in \mathbf{k} is also feasible.

In order to explore another direction in the multi-dimensional parameter space, one could consider replacing the atoms or molecules with artificial atoms such as quantum dots, which generate electronic bound states with a level structure similar to Figure 2(a). In this case, the dipole length ℓ_{atom} could be increased by several orders of magnitude, see Eq. (8), but the realistic energy and intensity scales would also have to be modified.

In summary, we have presented a scheme for entangling photons via double application of the quantum Zeno effect, which might be realizable with present day technology. Note that we consider the (pseudo) deterministic creation of entanglement between two *given* photons, which is very different from the *spontaneous* creation of entangled photon pairs via parametric down conversion. Our scheme is based on free-space linear optics in connection with a two-photon absorbing medium. Even though the example parameters above show that it might be hard to directly reach the error threshold of 10^{-4} or 10^{-5} required for universal quantum computation, the achievable success probabilities for entangling photons are already comparably high. In addition, by placing photon detectors around the apparatus, one can detect occurring errors with high probability and thus apply error correcting schemes; see, e.g., [7].

* ralf.schuetzhold@uni-due.de

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